SIGNED GRAPHS FROM PROPER COLORING OF GRAPHS

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ABSTRACT

Let $\chi(G)$ denote the chromatic number of a graph G. Under the proper coloring of a graph G with $\chi(G)$ colors, we define a signed graph from it. The obtained signed graph is defined as parity colored signed graph and denoted as S_c. The signs of edges of G are defined from the colors of the vertices as $+(-)$ if the colors on the adjacent vertices are of the same (opposite) parity. In this paper, we initiate a study on S_c. We further investigate the chromatic rna number of some classes of graphs concerning proper coloring.

KEYWORDS

Signed graph, parity colored signed graph of a graph, chromatic rna number.

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1. INTRODUCTION

In this paper, we consider simple connected graphs. The smallest positive integer k such that $f : V(G) \to \{1,2,\ldots,k\}$ such that $f(a) \neq f(b)$ whenever ab is an edge in G is called the chromatic number of G and it is denoted by $\chi(G)$.

In [7], the concept of a signed graph has been introduced. Let $S = (G, \sigma)$ be a signed graph with $\sigma : E(G) \to \{+, -\}$ the signature of G, where G is the underlying graph of S. The edges of G receiving $+(-)$ sign are called the positive (negative) edges of S. A signed graph is all positive (negative) if all the edges of S are positive (negative). A homogeneous signed graph is a signed graph in which either all the edges are positive or all negative and heterogeneous, otherwise. $E^-(S)\big(E^+(S)\big)$ denotes the negative (positive) edge set of the signed graph and $E(S) = E^-(S) \cup E^+(S)$ is the edge set. In [2], by the negation, we mean a signed graph $\eta(S)$ obtained S by reversing the sign of every edge S. By $d^-(v)\big(d^+(v)\big)$ we mean the number of negative (positive) edges incident to v and $d(v) = d^-(v) + d^+(v)$. The positive (negative) edges in S are represented by solid (dashed) line segments as shown in Figure 1. The negative section of a signed S is the maximal connected edge-induced subgraph in S consisting of only the negative edges S as defined in [2].

Motivated by the definition of rna number $\sigma^-(G)$[3], we initiate the concept of chromatic rna of G. For a detailed study of the rna number, we refer to [4,9–11]. The signed graphs $S = (G, \sigma)$ and $S' = (H, \sigma')$ are isomorphic if there exists a one-to-one correspondence between the vertex sets which preserves adjacency and signs on it.

A triangular Snake graph $TS(L)$ is obtained from a path on $L + 1$ vertices in which every edge is replaced by a triangle. The sign of a cycle (path) in a signed graph is the product of the signs of its edges. A cycle is said to be positive if the product of the signs of the edges is positive or the cycle has an even number of negative edges. A signed graph S is said to be balanced if all the cycles in S are positive [7]. Therefore, acyclic signed graphs are always balanced. Two vertices u and v of S_c are of the same parity if their colors $c(u)$ and $c(v)$ are both odd or both even and of opposite parity otherwise [4].

Motivated by the concept of set coloring in signed graphs [1] and induced signed graphs [5], we initiate a study on S_c of a graph. We refer to [6,8,12,13] for our study. Throughout the paper, by S_c we mean parity colored signed graph of a graph.

1.1. PRELIMINARIES

Definition 1.1. Let $A = \{1,2,\ldots,\chi(G)\}$ be a set of colors and $c : V(G) \to A$ be an onto function. Then parity colored signed graph of a graph G (S_c, in short) is defined by taking the signature function for every edge uv in G as:

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$$\sigma_c(uv) = \begin{cases} + \text{, } c(u) \text{ and } c(v) \text{ are both odd or both even} \\ - \text{, Otherwise.} \end{cases}$$

Example 1. In Figure 1(a) we show a proper coloring of a graph and in Figure 1(b) its S_c.

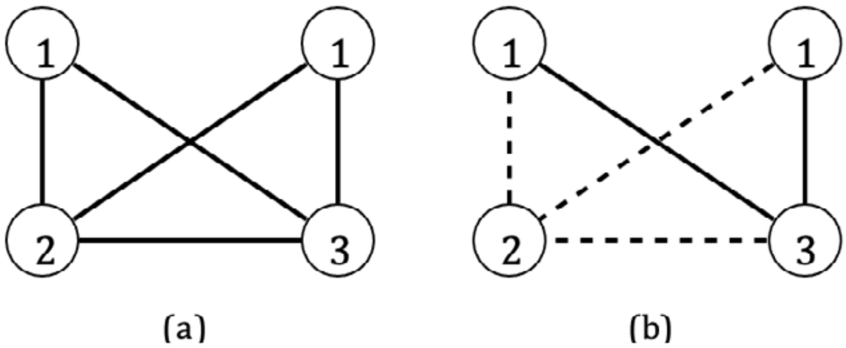

Figure 1. A graph with its Sc

Definition 1.2. The chromatic rna number of a graph G, denoted by $\sigma_c^-(G)$, is the smallest number of negative edges in S_c with respect to any proper coloring of G.

2. RESULTS ON Sc

Now we investigate some properties of S_c.

Observation 2.1. *S_c is positive homogeneous if $\forall v_i \in V(S_c), c(v_i) \equiv 1(mod2)$ $(c(v_i) \equiv 0(\mathrm{mod}2))$.*

From the definition of S_c, we can see that it is not unique. Does there exist a graph whose Sc is unique? The answer is yes as shown below.

Proposition 2.2. *S_c on a complete graph is unique up to isomorphism.*

Proof. The chromatic number of K_n is n with $c : V(K_n) \to \{1,2,\ldots,n\}$ as the vertex coloring. Since K_n is uniquely colored under c, there is exactly one S_c on K_n up to isomorphism.

Theorem 2.3. *Every non-trivial S_c of order n will have at least one negative edge.*

Proof. Let G be a non-trivial graph with $\chi(G) = k \leq n$ and $|V(G)| \geq 2$. Therefore, $\chi(G) = k \geq 2$. If $\chi(G) = 2$, then there exist at least two vertices colored with 1 and 2. Therefore, S_c will have a negative edge between these two vertices. So let $\chi(G) = k > 2$. Under the proper coloring of the graph, there exists a vertex v_i colored with m which is adjacent to vertices colored with $\{1,2,\ldots m-1, m+1,\ldots k\}$. If the vertex v_i is colored with an odd (even) number, then the edge between v_i and the vertex colored with $2(1)$ is a negative edge in S_c. Hence the result follows.

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The following theorem gives the balanced nature of S_c.

Theorem 2.4. *S_c of graph G is balanced.*

Proof. Consider S_c of G and let $o_i(e_i)$ represent the colors with odd (even) positive numbers. If G is acyclic, then obviously S_c is balanced. Assume that S_c contains at least one cycle. Consider an arbitrary cycle C_k in G. Without loss of generality, let C_k be the cycle on the vertices $v_1v_2\ldots v_kv_1$. Consider a path P_k on the vertices $v_1v_2\ldots v_k$ of the cycle C_k. Then, there are two cases:

Case 1: Under the function c, if the end vertices of P_k are colored with numbers of the same parity, then S_c of P_k will have an even number of negative edges, and the edge between v_1 and v_k will receive a positive sign. We can observe that when vertices of opposite colors are adjacent in a cycle, it will always induce two negative edges as seen in Figure 2. Therefore the cycle has an even number of negative edges.

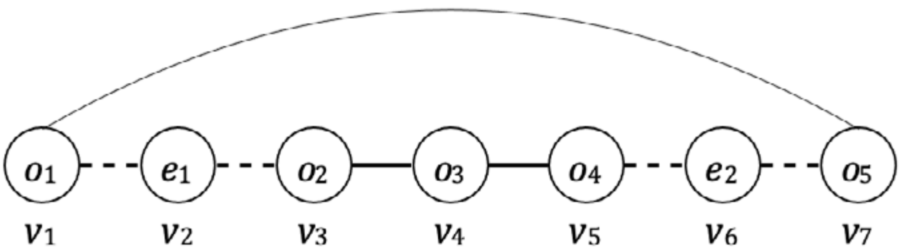

Figure 2. S_c on C_7

Case 2: Under the function c, if the end vertices of P_k are colored with numbers of opposite parity, then S_c of P_k will have an odd number of negative edges and the edge between v_1 and v_k will receive a negative sign as seen in Figure 3.

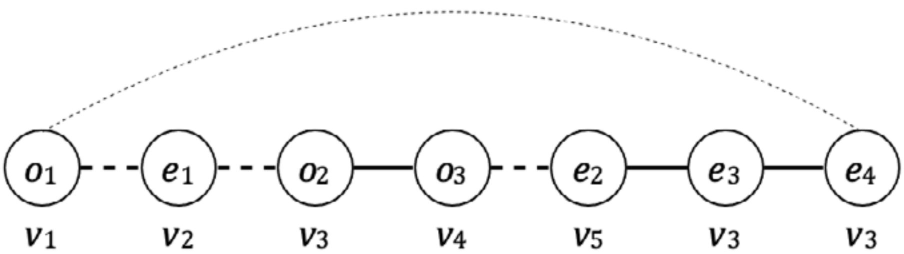

Figure 3. S_c on C_7

In both cases, any cycle in S_c has an even number of negative edges. Therefore, S_c is always balanced.

The converse need not be true as we know that positive homogeneous signed graphs are always balanced. However, S_c can never be a positive homogeneous signed graph. This observation leads to the next result.

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Theorem 2.5. *The negation of S_c is balanced if and only if the underlying graph of S_c is bipartite.*

Proof. Let S_c and $\eta\left(S_c\right)$ be the parity-colored signed graph and its negation respectively. Assume that $\eta\left(S_c\right)$ is balanced. Therefore, every cycle in $\eta\left(S_c\right)$ contains an even number of negative edges. This implies S_c contains an even number of positive edges. From Theorem 2.4, every cycle in S_c contains an even number of negative edges. Now, every cycle of S_c has an even number of negative and positive edges. Therefore, the underlying graph of S_c is bipartite.

Assume G is a bipartite graph. Therefore, every cycle in G is of even length. From Theorem 2.4, every cycle in S_c contains an even number of negative edges. Hence $\eta\left(S_c\right)$ is balanced.

Remark 1. *A subsigned graph of S_c need not be S_c.*

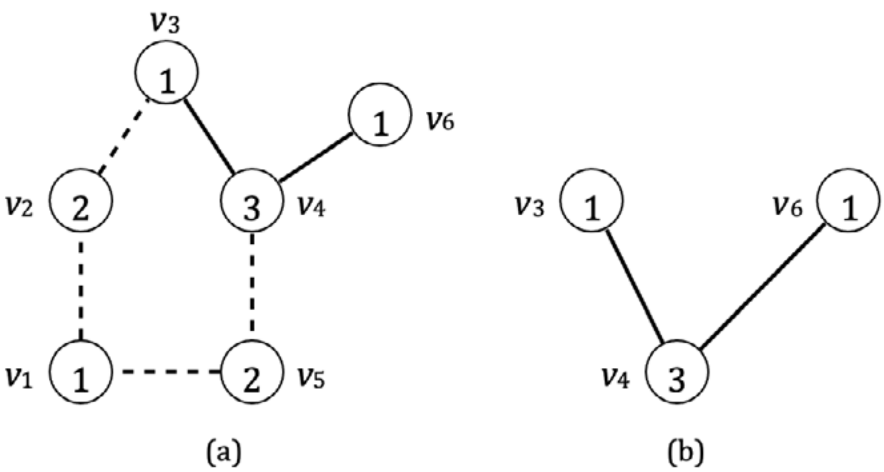

Figure 4

The underlying graph of the signed graph in Figure 4(a) has chromatic number 3. Therefore, the S_c in Figure 4(a) has at least one positive edge. Furthermore, the underlying graph of the signed graph in Figure 4(b) has chromatic number 2. Therefore, the S_c of Figure 4(b) is negative homogeneous. In Figure 4(a), the edges v_3v_4 and v_4v_6 will receive a positive sign. However, the subsigned graph with the same vertices will receive negative signs only since its chromatic number is 2. Therefore, the subsigned graph of S_c need not be S_c.

Remark 2. *The parity-colored signed graphs of graph G need not be isomorphic.*

The parity-colored signed graphs of Figure 5(a) are shown in Figure 5(b) and Figure 5(c). We can observe that they are not isomorphic to each other.

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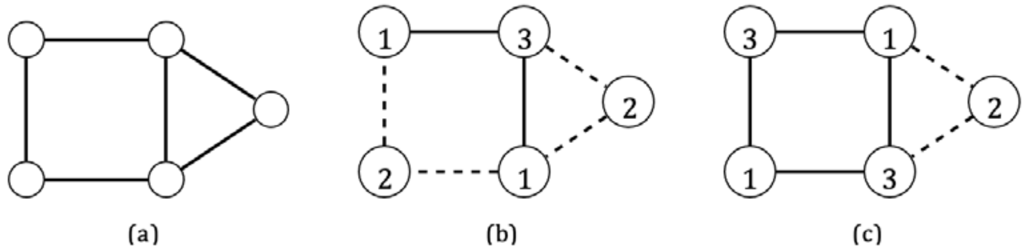

Figure 5

3. CHARACTERIZATION OF S_c

In this section, we will characterize S_c of some classes of graphs like bipartite graphs, cycles, and wheels. We also explore the 'chromatic rna' number of some classes of graphs concerning proper coloring.

We have already noted that there exists at least one negative edge in S_c. Therefore, it is impossible to have a positive homogeneous S_c. So we aim at finding a negative homogeneous S_c.

Theorem 3.1. *S_c is negatively homogeneous if and only if its underlying graph is bipartite.*

Proof. Assume that S_c is negatively homogeneous. From the balanced nature of S_c, its vertices can be partitioned into two subsets such that the edges between them are negative. Therefore, the vertices can be colored with two colors. This implies that the underlying graph of S_c is bipartite.

The converse is easy to see as G is bipartite and we need only two colors to color its vertices and get a negative homogeneous signed graph. Hence the result.

Corollary 3.2. *S_c of $G_1 \circ G_2$ is negative homogeneous if and only if G_2 is $\overline{K_n}$ and G_1 being $\overline{K_m}$ or bipartite graph.*

Corollary 3.3. *S_c of $G_1 + G_2$ is negative homogenous if and only if G_1 is $\overline{K_n}$ and G_2 being $\overline{K_m}$.*

We have seen that S_c is balanced. Next, we discuss the nature of a negative section in S_c of a cycle.

Proposition 3.4. *The negative section in S_c of a cycle C_k is always of even length or a whole cycle of even length.*

Proof. We know that $\chi\left(C_k\right) = 2(3)$, when k is even (odd). Therefore, there exists at least one vertex colored with 2 in C_k. In the proper coloring of C_k, the vertex colored with 2 is adjacent to the vertex colored with 1 or 3. This will give negative edges between the vertices colored with 1 and 2 or 2 and 3. Therefore, the negative

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section of C_k is of even length and when k is even the cycle will have all negative edges.

The next theorem gives the characterization of the signed cycle which is the S_c of its underlying graph.

Theorem 3.5. *A signed cycle C_k on k vertices is S_c if and only if C_k satisfies any one of the following:*

(i) C_k is negatively homogeneous for even k.

(ii) C_k is heterogeneous for odd k with length of each negative section even.

Proof. In C_k, the number of negative sections is equal to the number of positive sections. Let $l_{ni}\left(l_{pi}\right), 1 \leq i \leq \left\lfloor \frac{k}{2} \right\rfloor$, be the negative (positive) sections in a signed cycle.

Case 1: For even k, C_k is a bipartite graph. From Theorem 3.1, S_c of C_k is negative homogeneous. Hence (i) follows.

Case 2: For odd k, C_k is heterogeneous. From Proposition 3.4, the length of each negative section is even. Hence (ii) follows.

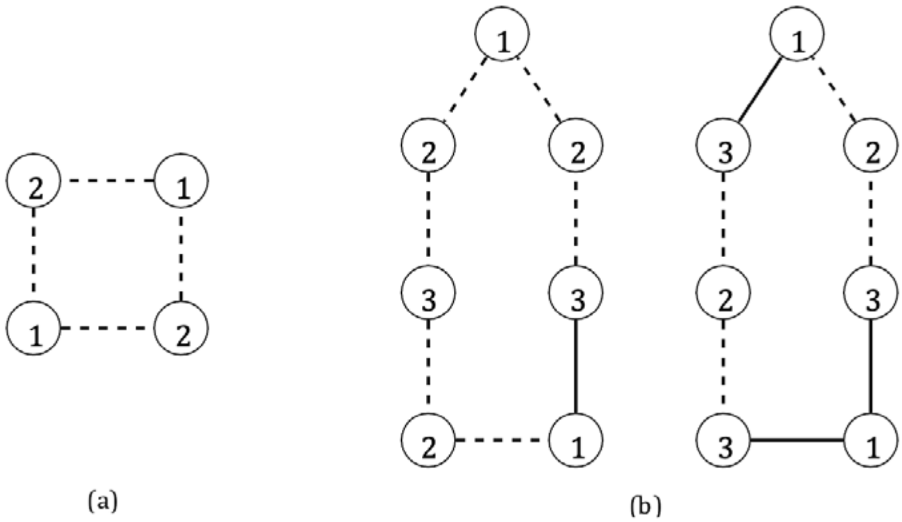

Figure 6. S_c of C_4 and C_7

Sufficiency is easy to see in Figure 6.

The next theorem gives the characterization of S_c on a wheel having an odd number of vertices.

Theorem 3.6. *A signed wheel $W_n = C_{n-1} + K_1$ for an odd integer n is S_c if and only if W_n satisfies any one of the following:*

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(i) C_n *is positive homogeneous and* $\left|E^-\left(W_n\right)\right| = \left|E^+\left(W_n\right)\right|$.

(ii) C_{n-1} *is negative homogeneous,* $\left\| E^-\left(W_n\right)\right| - \left|E^+\left(W_n\right) \right\| = n-1$ *and the distance between the vertices which are the end vertices of the positive edge lying on the* C_{n-1} *is two.*

Proof. We know that $\chi\left(W_n\right) = 3$ (n is odd). The vertices of C_{n-1} can be colored in two ways such that the edges of C_{n-1} are either positive (negative) homogeneous. If the vertices on the cycle are colored with 1 and 3 (1 and 2 or 2 and 3), then C_{n-1} is positive (negative) homogeneous respectively.

Case 1: If C_{n-1} is positive homogeneous, the vertex of K_1 will be colored with 2 only and the edges joining K_1 to all the vertices of C_{n-1} are negative.

Therefore, $\left|E^-\left(W_n\right)\right| = \left|E^+\left(W_n\right)\right|$. Thus (i) holds.

Case 2: If C_{n-1} is negative homogeneous, then the vertex of K_1 can be colored with 1 or 3. Then the edges joining K_1 to all the vertices of C_{n-1} are negative (positive) depending upon the colors on the C_{n-1} and it is easy to see that the distance between any two vertices lying on C_{n-1} which have a positive edge incident on them is two. We can observe that $\left|E^-\left(W_n\right)\right| = n-1+\dfrac{n-1}{2}$ and $\left|E^+\left(W_n\right)\right| = \dfrac{n-1}{2}$.

Therefore, $\left\| E^-\left(W_n\right)\right| - \left|E^+\left(W_n\right) \right\| = n-1$. Thus (ii) holds.

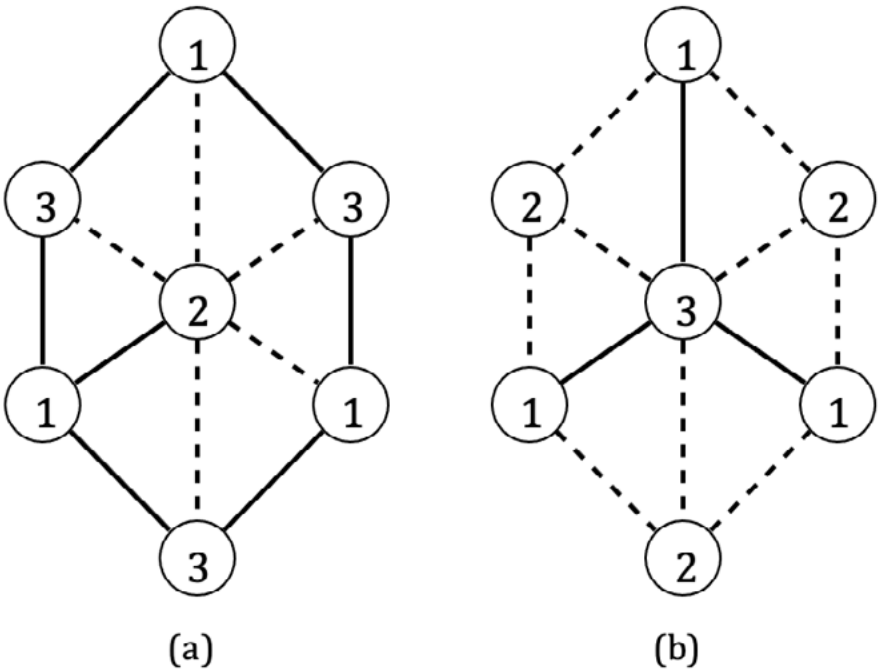

Figure 7. S_c of W_7

Sufficiency part is easy to see in Figure 7.

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Theorem 3.7. *A signed graph on $TS(L)$ is S_c if and only if the following conditions hold:*

(i) *Every cycle in $TS(L)$ has exactly two negative edges and*

(ii) *For a vertex $v \in V(TS(L))$ whose $d(v) = 4$, its $d^{-}(v)$ is either 4 or 2, and $d(v) = 2$ is due to two negative edges incident at v lying in two adjacent triangles.*

Proof. Assume that $TS(L)$ is S_c. We know that $\chi(TS(L)) = 3$. From the definition of S_c, each triangle of $TS(L)$ will have 2 negative edges. Hence (i) follows.

Now we prove (ii), let vertex $v \in V(TS(L))$ whose $d(v) = 4$ has $d^{+}(v) = 4$. That is, there is an edge in a triangle whose end vertices receive same color. This is a contradiction. Similarly we can show that $d^{+}(v) \neq 3$ and $d^{+}(v) \neq 1$. Therefore, $d^{+}(v)$ can be 2 or 0. In other words, $d^{-}(v) = 2$ or 4. If $d^{-}(v) = 2$ and two negative edges lie in the same triangle then the adjacent triangle will have exactly one negative edge, which is not possible. Thus $d^{-}(v) = 2$ is due to the negative edges lying in two adjacent triangles. When $d^{-}(v) = 4$, the proof is easy to see. Thus (ii) follows.

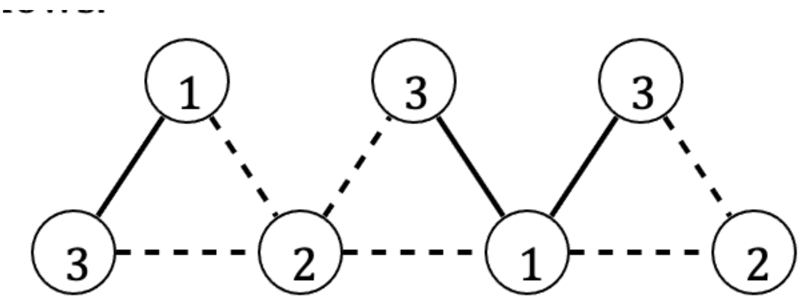

Figure 8. S_c of $TS(3)$

Further sufficiency is easy to see. S_c of $TS(3)$ is shown in Figure 8. Hence the proof.

4. CHROMATIC RNA NUMBER

Definition 4.1. The chromatic rna number of a graph G, denoted by $\sigma_c^{-}(G)$, is the smallest number of negative edges in S_c with respect to any proper coloring of G.

We investigate the 'chromatic rna' number of bipartite graphs, complete graphs, multipartite graphs, and cycle-related graphs.

Proposition 4.2. *For any bipartite graph B, $\sigma_c^{-}(B) = |E(B)|$.*

Proof. From Theorem 3.1, S_c of a bipartite graph is negative homogeneous. Therefore, $\sigma_c^{-}(B) = |E(B)|$.

Proposition 4.3. *For any complete graph Kn,*

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$$\sigma_c^-\left(K_n\right) = \begin{cases} \frac{n^2}{4}, n \text{ is even.} \\ \frac{n^2-1}{4}, n \text{ is odd} \end{cases}$$

Proof. From Proposition 2.2, the S_c of a complete graph, K_n is unique up to isomorphism. We know that $\chi\left(K_n\right) = n$. Let $X(Y)$ be the set of vertices colored with even (odd) positive integers from the set $\{1,2,\ldots,n\}$. The edges of K_n that occur across the set $X(Y)$ receive a negative sign. That is, the total number of negative edges is $|X||Y|$.

Case 1: When n is even, $|X| = |Y| = \frac{n}{2}$. Hence, $|X||Y| = \frac{n^2}{4}$.

Therefore, $\sigma_c^-\left(K_n\right) = \frac{n^2}{4}$.

Case 2: When n is odd, $|X| = \frac{n-1}{2}$, $|Y| = \frac{n+1}{2}$. Hence, $|X||Y| = \frac{n^2-1}{4}$.

Therefore, $\sigma_c^-\left(K_n\right) = \frac{n^2-1}{4}$. Hence de proof

Proposition 4.4. *For any complete r-partite graph* $K_{n,n,\ldots,n}$,

$$\sigma_c^-\left(K_{n,n,\ldots n}\right) = \begin{cases} \frac{r^2}{4}n^2, r \text{ is even} \\ \frac{r^2-1}{4}n^2, r \text{ is odd} \end{cases}$$

Proof. From Proposition 2.2, the Sc of a complete r-partite graph, $K_{n,n,\ldots,n}$ is unique up to isomorphism and its chromatic number is r. Let $X(Y)$ be the set of vertices colored with even (odd) positive integers from the set $\{1,2,\ldots,r\}$. The edges of $K_{n,n,\ldots,n}$ will receive a negative sign if and only if they occur between the sets X and Y. Since each set has n elements, the total number of negative edges is $|X||Y|n^2$.

Case 1: When r is even, $|X| = |Y| = \frac{r}{2}$. Hence, $|X||Y| = \frac{r^2}{4}$.

Therefore, $\sigma_c^-\left(K_{n,n,\ldots n}\right) = \frac{r^2}{4}n^2$.

Case 2: When r is odd, $|X| = \frac{r-1}{2}$, $|Y| = \frac{r+1}{2}$. Hence, $|X||Y| = \frac{r^2-1}{4}$.

Therefore, $\sigma_c^-\left(K_{n,n,\ldots nn}\right) \frac{r^2-1}{4}n^2$. Hence the proof.

We observe that the chromatic rna number of bipartite graphs, complete graphs, and complete multipartite graphs with respect to proper coloring is equal to the number of negative edges in them respectively. Next, we discuss a class of graphs for which it does not hold true.

Proposition 4.5. *For any cycle* C_k,

$$\sigma_c^-\left(C_k\right) = \begin{cases} k, k \text{ is even.} \\ 2, k \text{ is odd.} \end{cases}$$

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Proof. For any cycle C_k, we have the following two cases:

Case 1: The cycle on the even number of vertices is a bipartite graph. From Theorem 3.5, the S_c of the cycle is negatively homogeneous.

Therefore, $\sigma_c^-\left(C_k\right) = k$ (k is even).

Case 2: Let the vertices of the cycle be vi such that $v_iv_{i+1} \in E\left(C_k\right)$ and $v_kv_1 \in E\left(C_k\right) \forall i, 1 \leq i < k$. From Theorem 2.3 and Theorem 2.4, C_k is balanced, and C_k will have at least one negative edge. Therefore, at least two negative edges exist in C_k. Let the edges be v_1v_k, and $v_k - 1v_k$. Since the chromatic number of the odd cycle is three, let $C: V\left(C_k\right) \to \{1,2,3\}$ be the vertex coloring function. Consider the following coloring.

The vertices $v_1, v_3, \ldots v_{k-2}$ are colored with 1. The vertices $v_2, v_4, \ldots v_{k-1}$ are colored with 3, and the vertex v_k is colored with 2. This coloring gives two negative edges in the C_k. Therefore, $\sigma_c^-\left(C_k\right) = 2$ (k is odd).

Proposition 4.6. *Le G is a graph having k cycles with exactly one vertex in common and the length of each cycle is greater than or equal to 3. If m cycles are of odd length and the remaining cycles are of even length then,*

$$\sigma_c^-(G) = \begin{cases} |E(G)|, & \text{If all cycles are even.} \\ 2k, & \text{If all cycles are odd.} \\ 2m, & \text{otherwise} \end{cases}$$

Proof. Consider a graph G having k cycles with exactly one vertex in common. We arrange the cycles in such a way that the first m cycles are of odd length and the remaining cycles are of even length.

Case 1: If G contains cycles only on an even number of vertices, then G is a bipartite graph. From Theorem 3.5, $\sigma_c^-(G) = |E(G)|$.

Case 2: If G contains cycles only on the odd number of vertices, then $\chi(G) = 3$. From Theorem 3.5, cycles on the odd number of vertices will have a minimum of two negative edges. From Proposition 4.5, $\sigma_c^-(G) = 2k$.

Case 3: If G contains cycles on odd and even numbers of vertices, then clearly $\chi(G) = 3$. In this case cycles on even number vertices can be colored with 1 and 3 such that they have zero negative edges. From Proposition 4.5, cycles on the odd number of vertices will have a minimum of two negative edges. Therefore, $\sigma_c^-(G) = 2m$. Hence the proof.

Proposition 4.7. *For a triangular snake graph $TS(L), \sigma_c^-(TS(L)) = 2L$.*

The following result gives the characterization of a graph with respect to a specific rna number associated with it.

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Proposition 4.8. *$\sigma_c^-(G) = 1$ if and only if G is P_2.*

We know that there exists more than one graph whose rna number is $n \in \mathbb{N}$, for $n \geq 2$. Is it possible to construct S_c with a given rna number? The answer is yes.

Theorem 4.9. *There exists S_c for every natural number n such that $\sigma_c^-(G) = n$.*

Proof. For every n ∈ N, consider a path P_{n+1} on $n+1$ vertices. From Theorem 3.1, the path is always negative homogeneous. Therefore, there exists S_c with a given rna number.

We also have some other graphs such as $K_{1,n}$ with fixed rna numbers.

5. CONCLUSION

In the paper, we have initiated a study on S_c. We have given the characterization of S_c on some classes of graphs. We have investigated the rna number with respect to the proper coloring of some classes of graphs and we have also shown the existence of graphs with a given rna number.

6. ACKNOWLEDGMENT

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