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ON CONVERGENCE OF ITERATIVE METHOD FOR DETERMINATION OF WEIBULL PARAMETERS BY MAXIMUM LIKELIHOOD METHOD

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ABSTRACT

The Weibull distribution is frequently used for the assessment of wind energy potential and modeling of wind speed data. The parameters of Weibull distribution are determined by a number of methods; Maximum Likelihood Methods is one of them. The values of scale and shape parameters of Weibull distribution are found by the help of Maximum Likelihood function. Two different techniques are used to find the parameters. One is known as iterative method, in which a start value of 'k' is set and iterations are terminated when given criterion is reached. The second method is Newton Raphson method of finding roots. We report here a problem of non-convergence of iterative method. We suggest the Newton Raphson method as the best choice for finding the value of 'k' through Maximum Likelihood Method.

KEYWORDS

Weibull distribution, Weibull parameter, Maximum Likelihood Method.

1. INTRODUCTION

We are living in machine era; people, at work place have been replaced by machines or robots. At home too, daily routine works are done by electronic devices. The use of electricity, have increased drastically in last four decades. Fast depletion of fossil fuels has made people around the world to think for alternate source of energy. Uninterrupted, cost effective, and environmental friendly source of energy is a dream and desire of today's world. Wind energy is a good choice as an alternate source of energy. Many parts of world have got excellent potential of wind speed; wind energy is rapidly growing as a source of energy around the world [1, 2]. Most of the countries have been generating electrical energy through wind [3].

Wind fluctuates time to time, the fluctuations also depends on the height from the sea level. Large amplitude fluctuations are the challenges in designing and installing wind farms [4-6]. The planning, designing, installing and operating wind turbines depends on wind potential and its characteristics [7].

The modeling of wind power plays in important role in assessing wind potentials [8]; different statistical distributions and mathematical techniques have been employed to model wind data [9]. Most widely used statistical distribution to model wind data is Weibull distribution [10]. There are different forms of Weibull distribution depending upon no. of parameters. The simplest Weibull distribution has two parameters; its Probability Density Function (PDF) is given in eq. (1)

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^k}, \qquad (1)$$

Here 'k' is known as shape parameter and 'c' is known as scale parameter. The cumulative distribution function (CDF) is given by eq. (2):

$$F(v) = 1 - e^{-\left(\frac{v}{c}\right)^k} \qquad (2)$$

2. ANALYSIS

Various statistical and mathematical methods are employed to find parameters 'k' and 'c'. Among them are Methods of moment, Empirical Method, Energy Pattern Factor Method, Graphic Method, Least Square Method, Equivalent Energy

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Method, Maximum Likelihood Method, and Modified Maximum Likelihood Method. In this study we considered only Maximum Likelihood Method. The values of Weibull parameters by this method are given by equations (3) and (4); 'k' and 'c' are found by iterative method or by Newton Raphson Method.

2.1. Iterative Method

In iterative method a start value of 'k' is selected and wind speed data is used to calculate sums in eq. (3), since sum of Logarithm of wind speeds is needed in the calculation, hence zero wind speeds are neglected in this method. The new value of 'k' is generated through eq. (3) and used in next iteration, the process continues until a given criterion is reached.

$$k = \left[\frac{\sum_i^n v_i^k ln(v_i)}{\sum_i^n v_i^k} - \frac{\sum_i^n ln(v_i)}{n}\right]^{-1} \qquad (3)$$

$$c = \frac{\sum_i^n v_i^k}{n} \qquad (4)$$

2.2. Newton Raphson Method

The eq. (5) obtained by differentiating Logarithm of Likelihood function with respect to shape parameter 'k' is used as a function of 'k' in Newton Raphson Method of finding roots.

$$f(k) = \frac{n}{k} - nlnc + \sum_{i=1}^{n} ln(v_i) - \sum_{i=1}^{n} \left(\frac{v_i}{c}\right)^k ln\left(\frac{v_i}{c}\right) = 0 \qquad (5)$$

An initial value of 'k' is selected as a starting point of Newton's method. Wind data of two coastal regions of Pakistan, namely, Gwadar and Ormara are used to calculate Weibull parameters by Maximum Likelihood Method. To investigate any dependence of calculation method (iterative and Newton's method) on start value of 'k'; various start values of 'k' are employed to calculate parameters. A dependence on start value of 'k' is found and shown in figures (1-3).

The iterative method is easy to implement but convergence is not guaranteed. Table 1 gives the results of iterative method with various start values of 'k'. The iteration oscillates between two values and does not converge.

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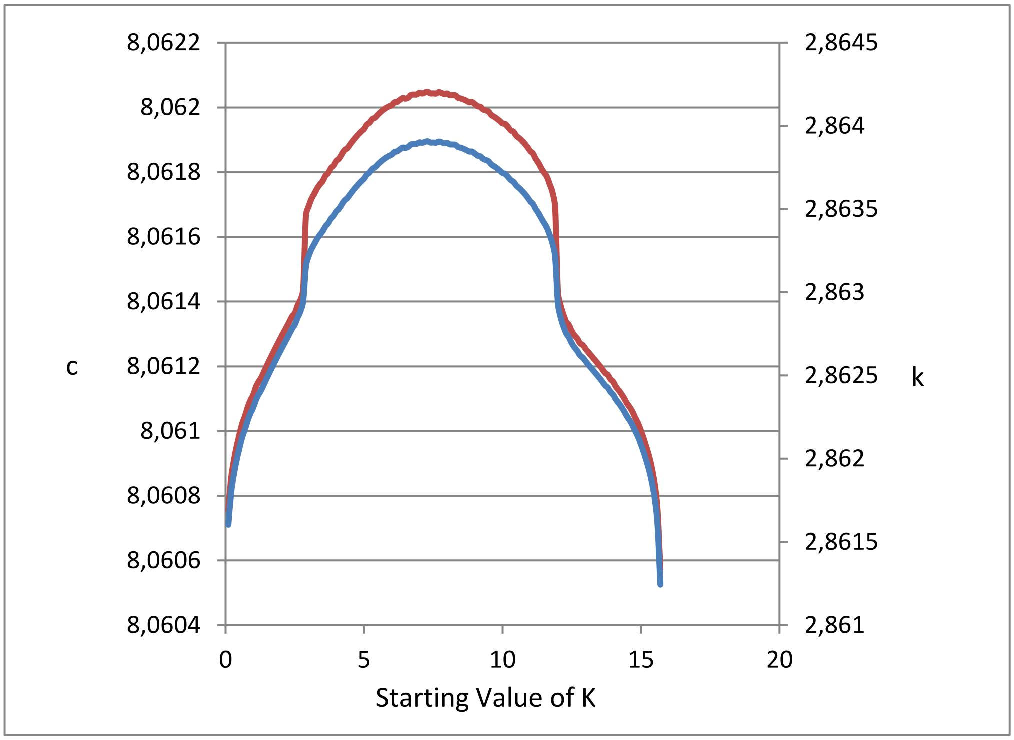

Figure 1. Variation of k and c with various start values of 'k' for Gwadar's wind data of Jan 2002.

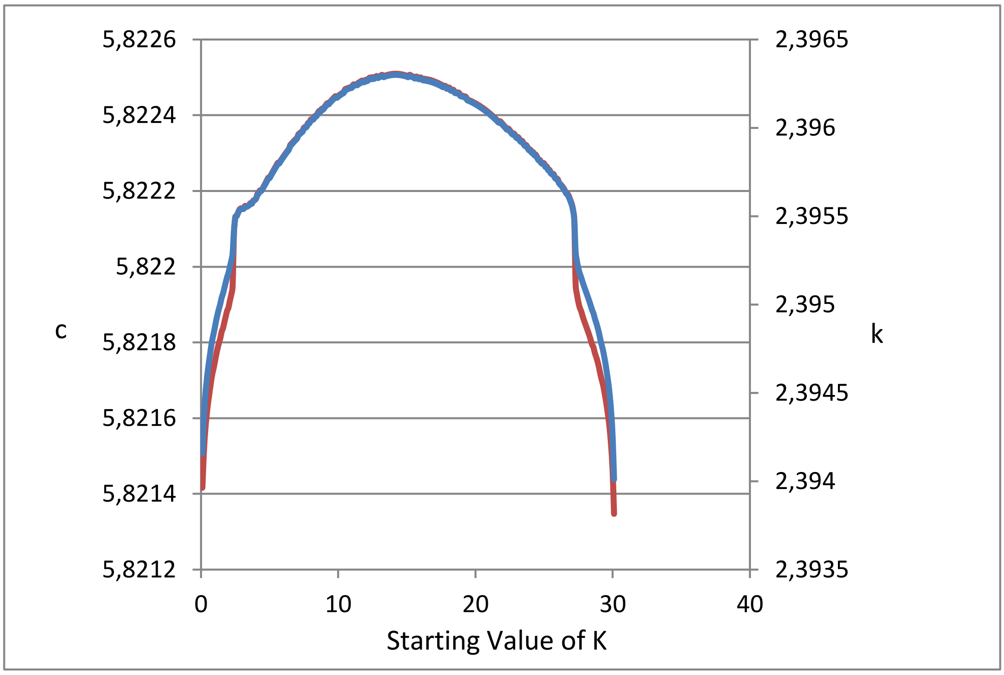

Figure 2. Variation of k and c with various start values of 'k' for Ormara's wind data of May.

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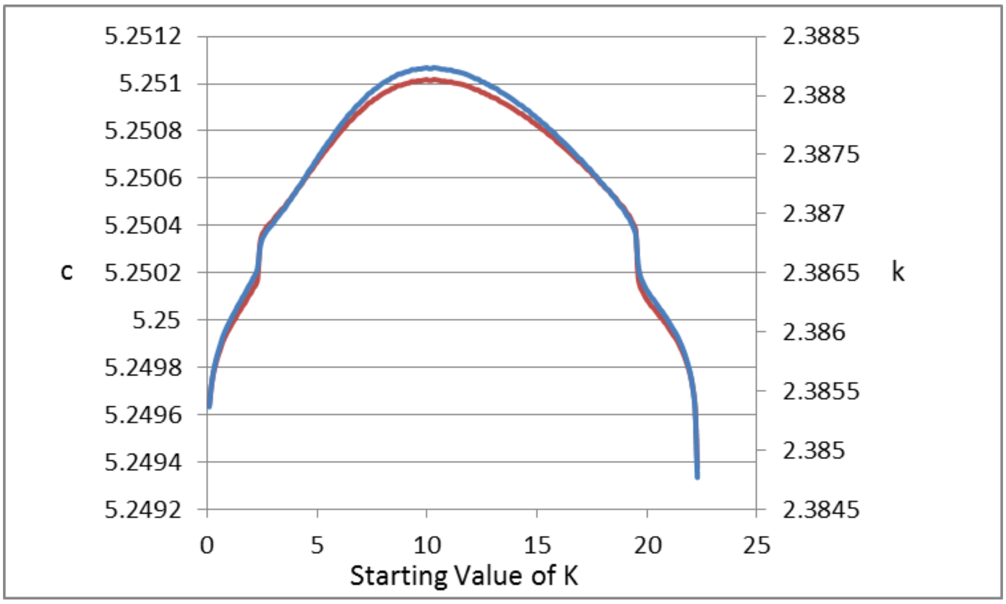

Figure 3. Variation of k and c with various start values of 'k' for Ormara's wind data of July.

3. CONCLUSION

Wind data of Gwadar (Jan 2002) and ten years data of Ormara have been used in this study. In figs (1-3) the results of Newton Raphson method are plotted. Various start values (starting from 0.1 and step size of 0.1) are taken to find Weibull distribution parameters. It is found that the values of scale and shape parameters increase with increasing start value of 'k'. Both approach to a maximum value at some start value of 'k'. If the start value further increased the shape and scale parameters start decreasing. The variation in shape parameter is less than 0.003 and in scale parameter it is less than 0.005 m/s.

Iterative method for finding Weibull parameters through Maximum Likelihood method was used for Gwadar. Various start values of 'k' were taken to find the parameters. It was found that the iterative method does not converge; the shape parameter 'k' oscillates between two values. One value generates the other, and convergence criterion does not approach. Table I shows the results of iterative method for start values of k = 1, 2, 3, 4, 5, 10, 20, 50. It can be seen that the iterations oscillate between two values 1.559878 and 5.466348. Hence, it is likely that the iterative method would not converge. It is suggested that if Weibull parameters are determined by Maximum Likelihood Method, preference to Newton Raphson method should be given over to Iterative method discussed above.

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Table 1. The results of iterative method for start value of k = 1, 2, 3, 4, 5, 10, 20, and 50.

	k = 1		k = 2		k = 3		k = 4	
Iteration #	K(i)	K(i+1)	K(i)	K(i+1)	K(i)	K(i+1)	K(i)	K(i+1)
1	1	8.706837	2	4.197874	3	2.725345	4	2.035211
2	8.706837	1.250435	4.197874	1.944241	2.725345	3.016747	2.035211	4.120408
3	1.250435	6.897759	1.944241	4.326401	3.016747	2.70944	4.120408	1.978533
4	6.897759	1.358366	4.326401	1.890822	2.70944	3.035577	1.978533	4.246483
5	1.358366	6.324136	1.890822	4.456776	3.035577	2.691786	4.246483	1.923543
6	6.324136	1.420669	4.456776	1.840735	2.691786	3.056757	1.923543	4.376043
7	1.420669	6.032818	1.840735	4.586002	3.056757	2.672215	4.376043	1.871283
8	6.032818	1.460676	4.586002	1.794821	2.672215	3.080584	1.871283	4.506351
9	1.460676	5.858908	1.794821	4.710889	3.080584	2.650554	4.506351	1.822699
10	5.858908	1.487841	4.710889	1.753689	2.650554	3.107394	1.822699	4.634299
20	5.548741	1.543403	5.185799	1.622172	2.504601	3.300865	1.652229	5.143535
30	5.486207	1.555836	5.385676	1.576777	2.284046	3.642588	1.585917	5.371494
40	5.471277	1.558871	5.445342	1.564204	2.005357	4.185911	1.566622	5.441479
50	5.46758	1.559626	5.461033	1.560968	1.757363	4.817671	1.561582	5.460044
60	5.466657	1.559815	5.465013	1.560152	1.623655	5.239461	1.560306	5.464764
70	5.466426	1.559862	5.466014	1.559947	1.577212	5.402853	1.559986	5.465951
80	5.466368	1.559874	5.466264	1.559895	1.564318	5.449951	1.559905	5.466249
90	5.466353	1.559877	5.466327	1.559883	1.560997	5.462208	1.559885	5.466323
100	5.466349	1.559878	5.466343	1.559879	1.560159	5.465309	1.55988	5.466342
1000	5.466348	1.559878	5.466348	1.559878	1.559878	5.466348	1.559878	5.466348
1001	1.559878	5.466348	1.559878	5.466348	5.466348	1.559878	5.466348	1.559878

	k = 5		k = 10		k = 20		k = 50	
Iteration #	K(i)	K(i+1)	K(i)	K(i+1)	K(i)	K(i+1)	K(i)	K(i+1)
1	5	1.669334	10	1.176412	20	1.170701	50	1.169289
2	1.669334	5.087693	1.215742	7.352177	1.170701	7.389628	1.169289	7.398942
3	5.087693	1.646441	7.10383	1.321036	7.389628	1.318344	7.398942	1.317682
4	1.646441	5.162693	1.340311	6.511902	1.318344	6.525857	1.317682	6.529293
5	5.162693	1.627765	6.413642	1.398092	6.525857	1.396505	6.529293	1.396116
6	1.627765	5.225456	1.409616	6.135374	1.396505	6.142708	1.396116	6.144508
7	5.225456	1.612746	6.082613	1.445849	6.142708	1.444821	6.144508	1.444569
8	1.612746	5.27699	1.453371	5.922233	1.444821	5.926673	1.444569	5.927761
9	5.27699	1.600814	5.889944	1.477642	5.926673	1.476941	5.927761	1.476769
10	1.600814	5.318627	1.482798	5.788011	1.476941	5.79091	1.476769	5.79162
20	1.570683	5.426604	1.542361	5.535962	1.541167	5.5365	1.541133	5.536632
30	1.562623	5.456201	1.555586	5.483227	1.555301	5.483353	1.555293	5.483384
40	1.560568	5.463794	1.558809	5.470543	1.558738	5.470574	1.558736	5.470582
50	1.560051	5.465708	1.559611	5.467397	1.559593	5.467405	1.559593	5.467407
60	1.559922	5.466188	1.559811	5.466611	1.559807	5.466613	1.559807	5.466613
70	1.559889	5.466308	1.559862	5.466414	1.55986	5.466415	1.55986	5.466415
80	1.559881	5.466338	1.559874	5.466365	1.559874	5.466365	1.559874	5.466365
90	1.559879	5.466346	1.559877	5.466352	1.559877	5.466352	1.559877	5.466352
100	1.559878	5.466348	1.559878	5.466349	1.559878	5.466349	1.559878	5.466349
1000	1.559878	5.466348	1.559878	5.466348	1.559878	5.466348	1.559878	5.466348
1001	5.466348	1.559878	5.466348	1.559878	5.466348	1.559878	5.466348	1.559878

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